Demand-oblivious routing: distributed vs. centralized approaches

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Introduction

Routing optimization is hard without a good traffic matrix

Rate-adaptive routing: adapt routing to the actual demands

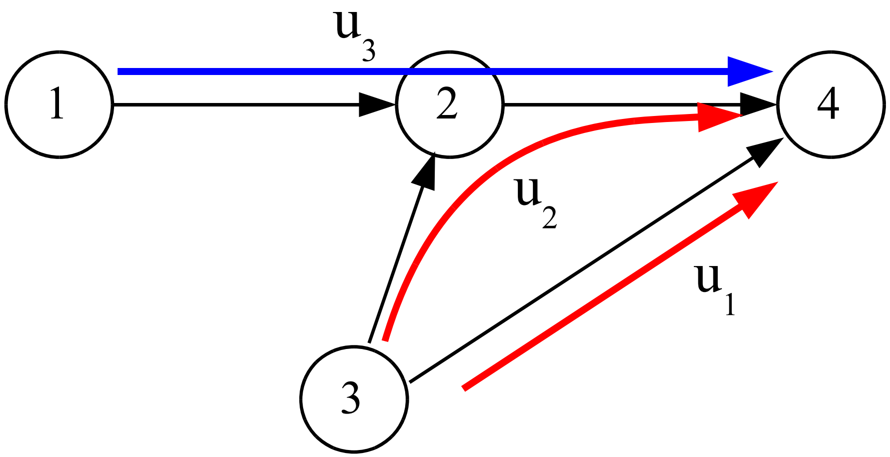
Build on demand-oblivious routing and play out the “distributed-centralized” trade-off

Our main tool: network geometry

Network geometry

Associate geometric objects with capacitated networks

Infer interesting properties

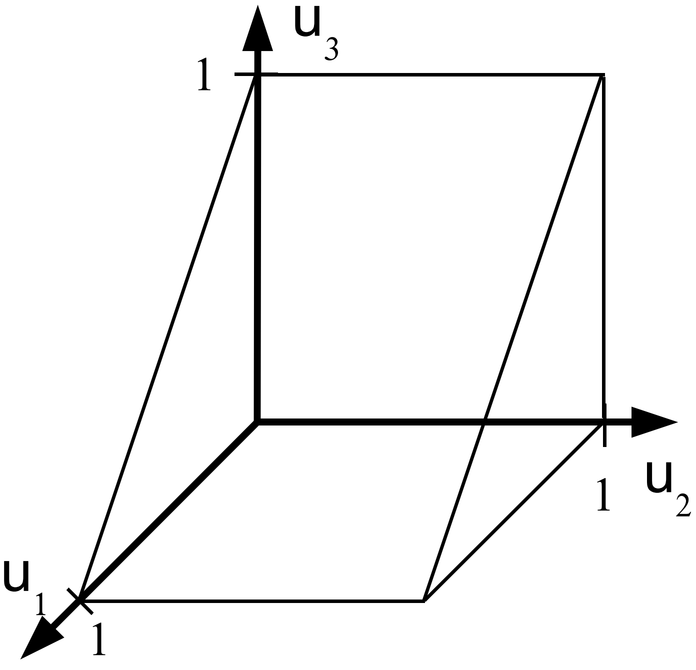


$$(s_1, d_1) = (3, 4)$$
$$(s_2, d_2) = (1, 4)$$

The flow polytope

The set of legitimate routings

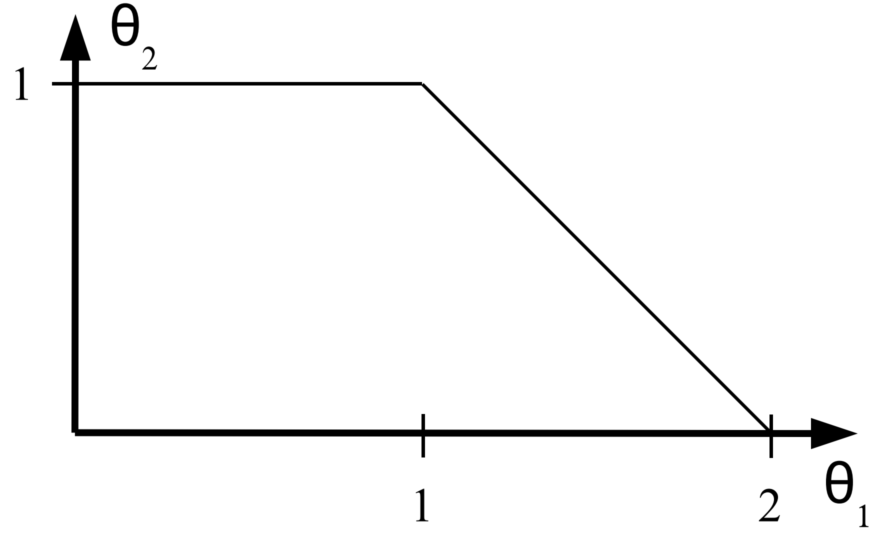
More precisely, the set of path-flows u the network can accommodate, subject to link capacities



The throughput polytope

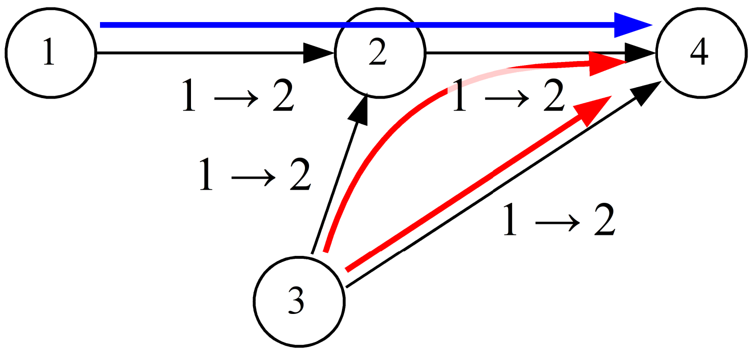
The set of admissible traffic matrices

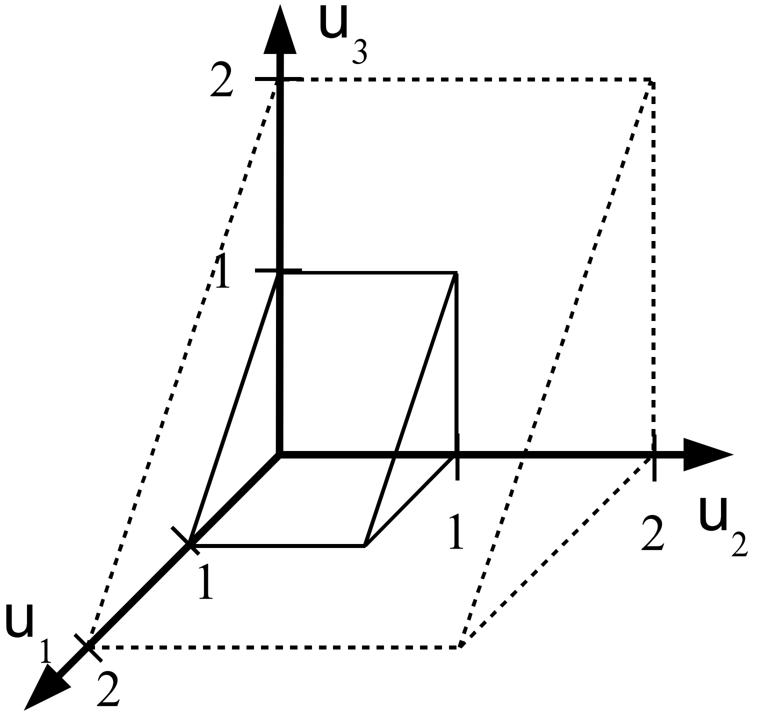
More precisely, the set of aggregate flows θ realizable in the network, subject to link capacities



Capacity scaling

Scaling the link capacities equals scalar multiplying the corresponding polytopes





Rate-adaptive routing

Adjust path flows according to actual user demands

A routing function tells how to map a traffic matrix to path-flows

$$u = \mathcal{S}(\theta)$$

We only treat affine routing functions

$$u = F\theta + g$$

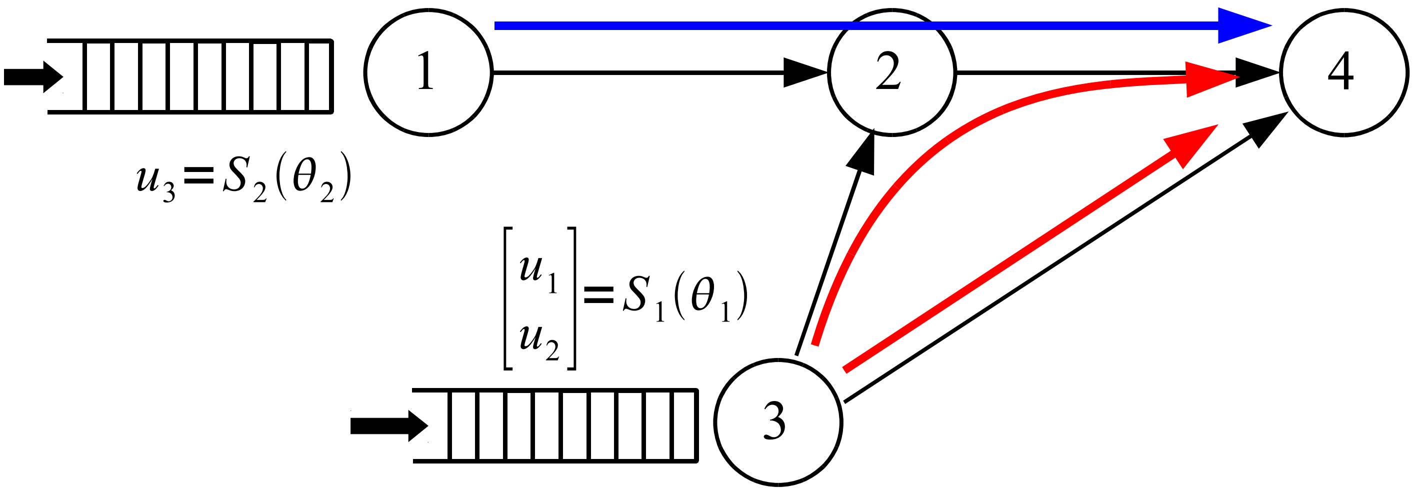
where F is a matrix and g is a constant transposition

For the kth user: $u_k = \mathcal{S}_k(\theta) = F_k\theta + g_k$

Already broad enough to describe single path routing, ECMP, oblivious routing, and many more

Adaptive routing: distributed model

The flow sent to a path depends on local information exclusively



$\mathcal{S}$ is distributed if $\frac{\partial \mathcal{S}_k}{\partial \theta_l} = 0$ wherever $k \neq l$

Demand-oblivious routing

Use the same set of traffic splitting ratios without respect to the traffic matrix

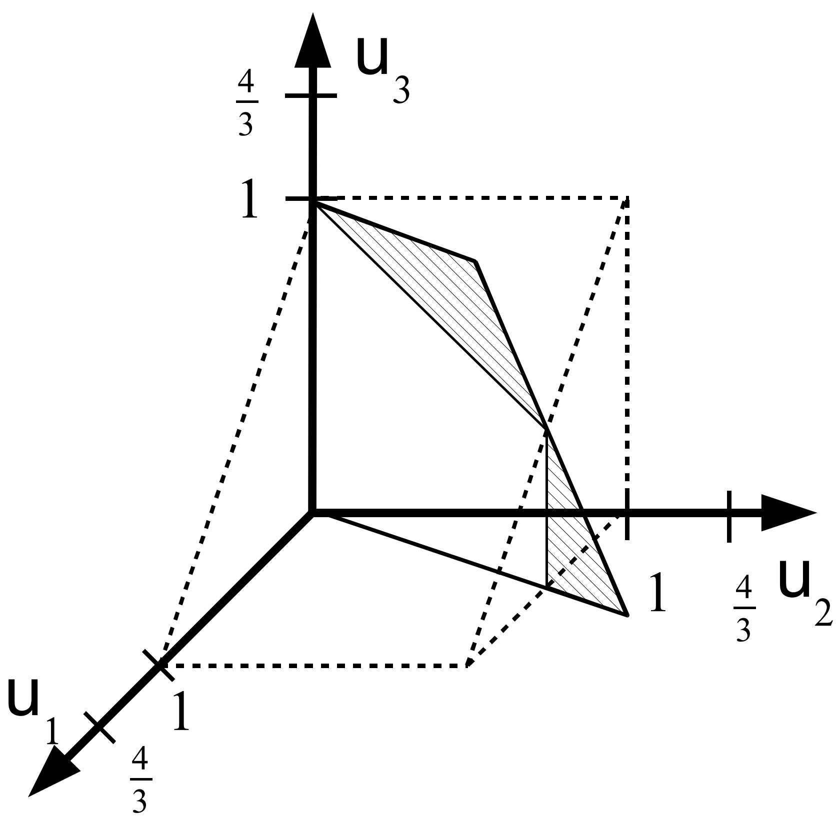
Choose the one that minimizes the link over-utilization experienced over any admissible traffic matrix

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 \\ \frac{2}{3} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Distributed and semi-static, so reasonably scalable

The problem with oblivious routing

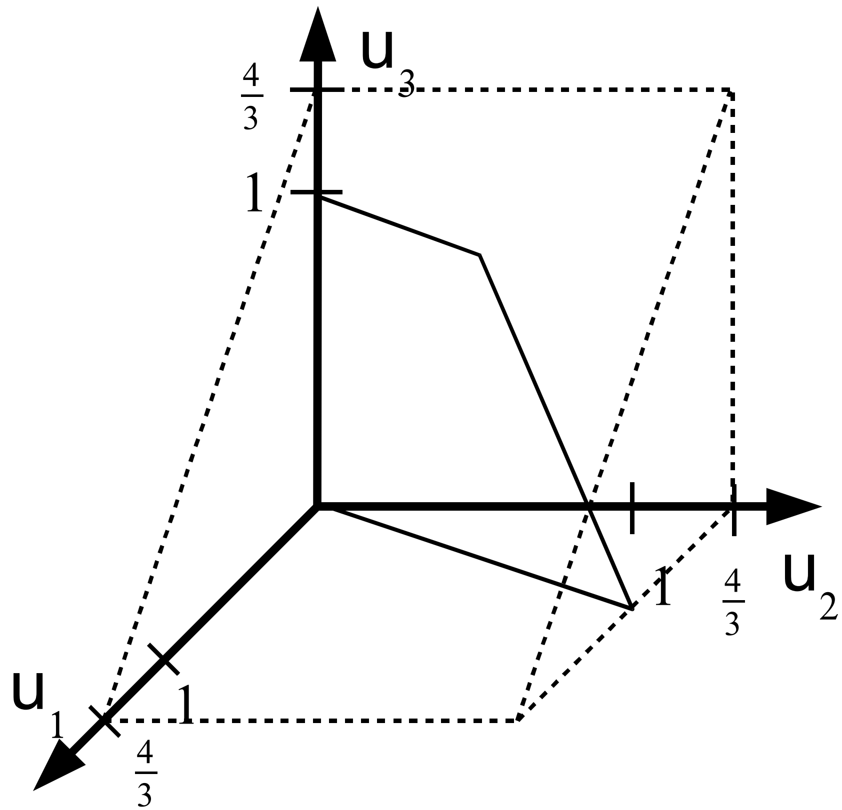
An oblivious routing function might order infeasible routing to some admissible traffic matrices



A geometric interpretation

Scale the flow polytope M up until it eventually contains all the possible path flows $\mathcal{S}(T)$

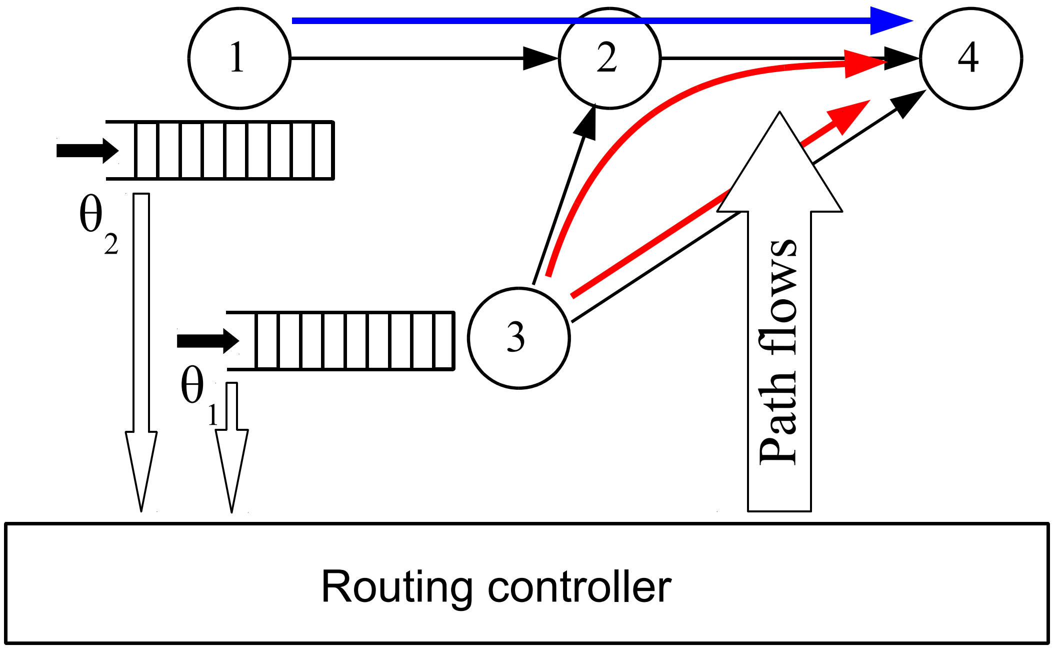
$$\min \alpha : \mathcal{S}(T) \subseteq \alpha M$$



Adaptive routing: centralized model

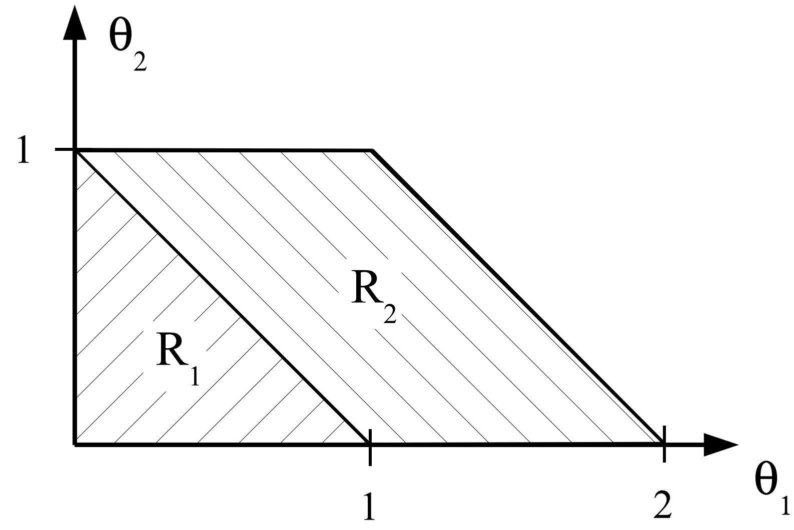
Let the routing function depend on global information

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$



Compound routing functions

Associate different routings to different regions of the throughput polytope: $\mathcal{S} = \{(R^i, \mathcal{S}^i) : i \in \mathcal{I}\}$



R_1 : if $\theta_1 + \theta_2 \leq 1$ then

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

R_2 : if $\theta_1 + \theta_2 \geq 1$ then

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Compound, centralized routing functions

Theorem: for any network, there is a continuous, compound, centralized affine routing function that can route any admissible traffic matrix without link over-utilization

Distributed:

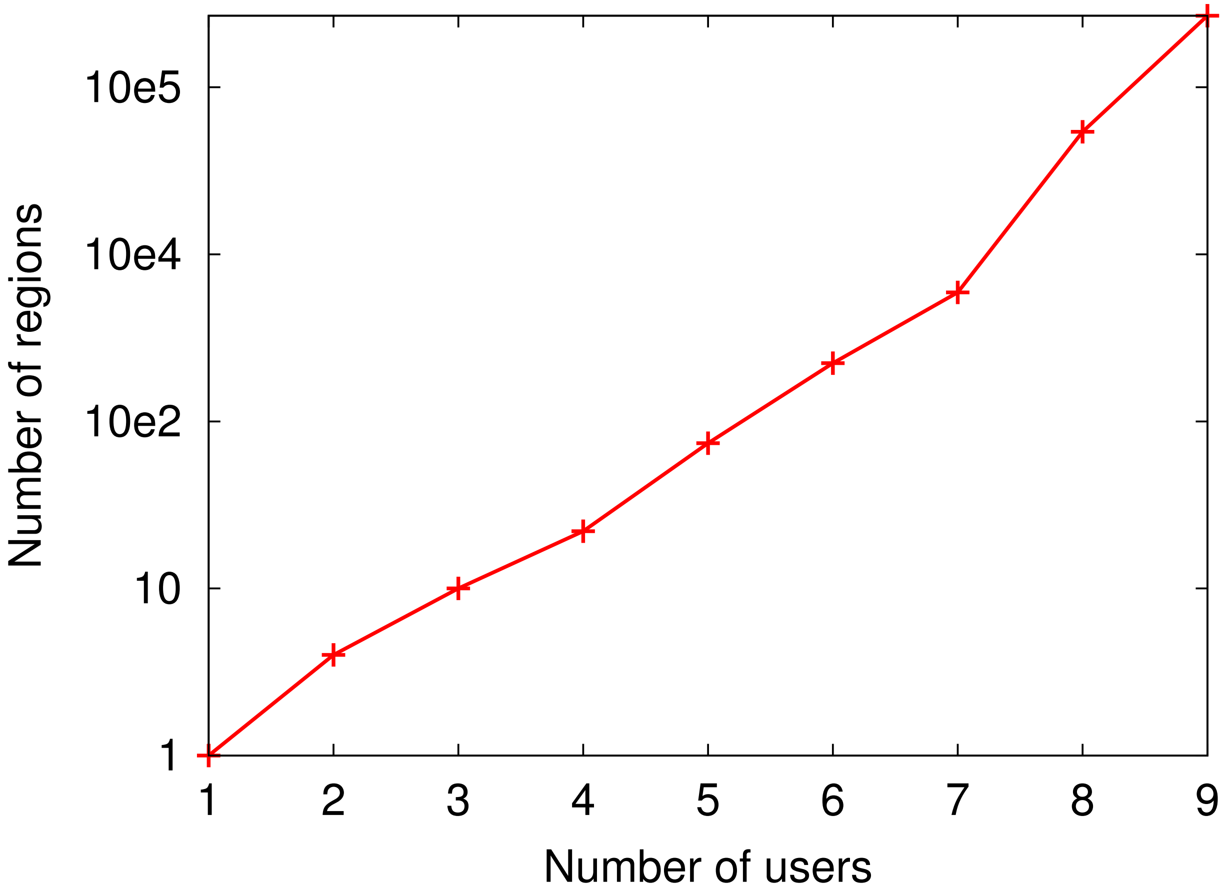
- Simple
- Scalable
- But inefficient

Centralized:

- Stable
- Feasible
- Optimizable
- Not quite scalable

Scalability of centralized adaptive routing

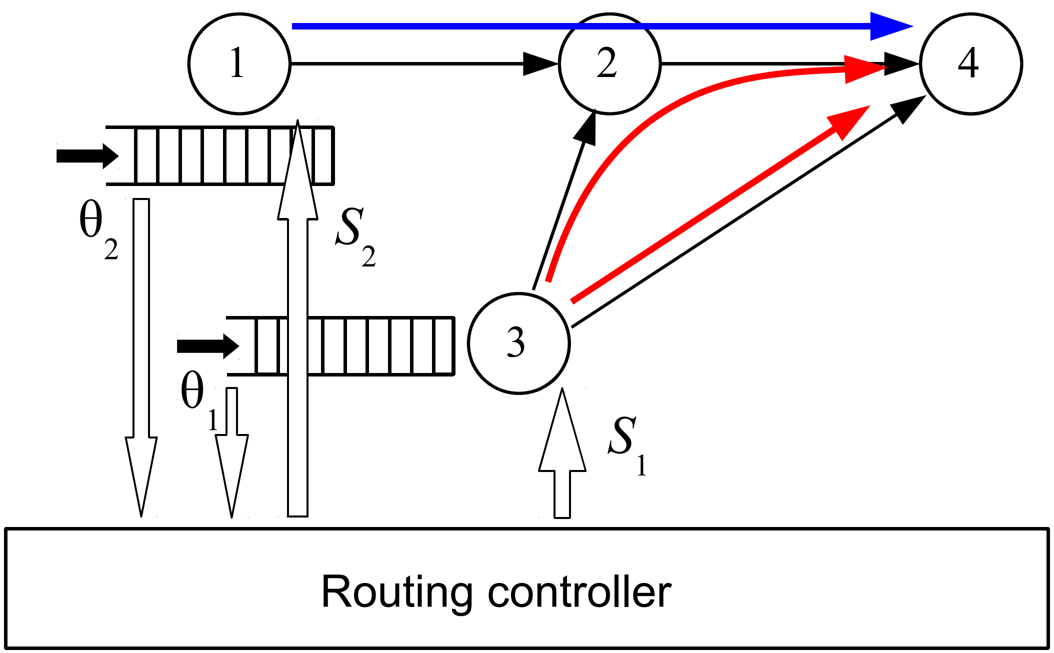
The number of regions and routing functions needed for optimal adaptive routing usually increases exponentially with the complexity of the network



Hybrid centralized-distributed model

The central controller computes $\mathcal{S} = \{(R^i, \mathcal{S}^i) : i \in \mathcal{I}\}$, where individual routing functions $\mathcal{S}^i$ are distributed

Observes the actual traffic matrix θ, chooses the region $\theta \in R_i$ and downloads the corresponding $\mathcal{S}^i$ to the routers



Hybrid oblivious routing algorithm

HYBRID_OBLIVIOUS_ROUTING(T)

function HYBRID_OBLIVIOUS_ROUTING(X)

    Compute an oblivious routing function $\mathcal{S}$ for X

    if α falls beyond some configured limit **then**

        store $\mathcal{S}$ and **return**

    end if

    $(k, t_k) \leftarrow$BEST_CUT(X)

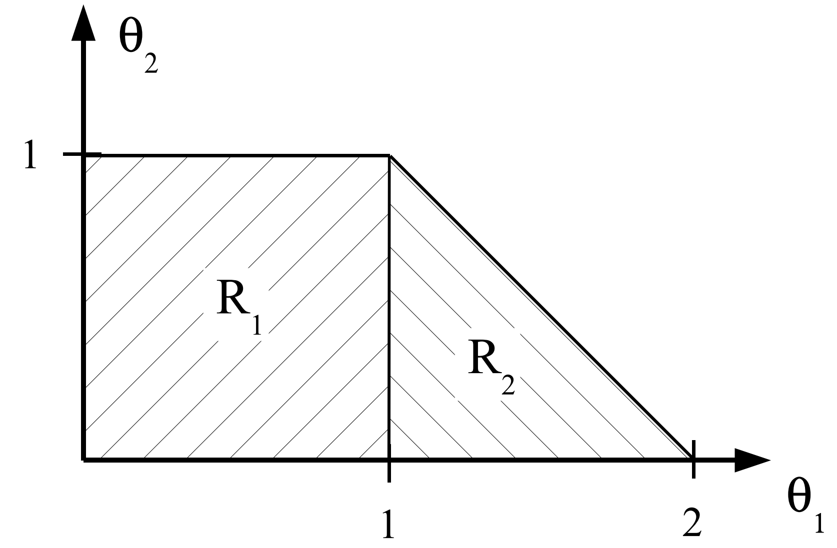
    HYBRID_OBLIVIOUS_ROUTING($X \cap T \cap \{\theta : \theta_k \leq t_k\}$)

    HYBRID_OBLIVIOUS_ROUTING($X \cap T \cap \{\theta : \theta_k \geq t_k\}$)

end function

Hybrid oblivious routing algorithm

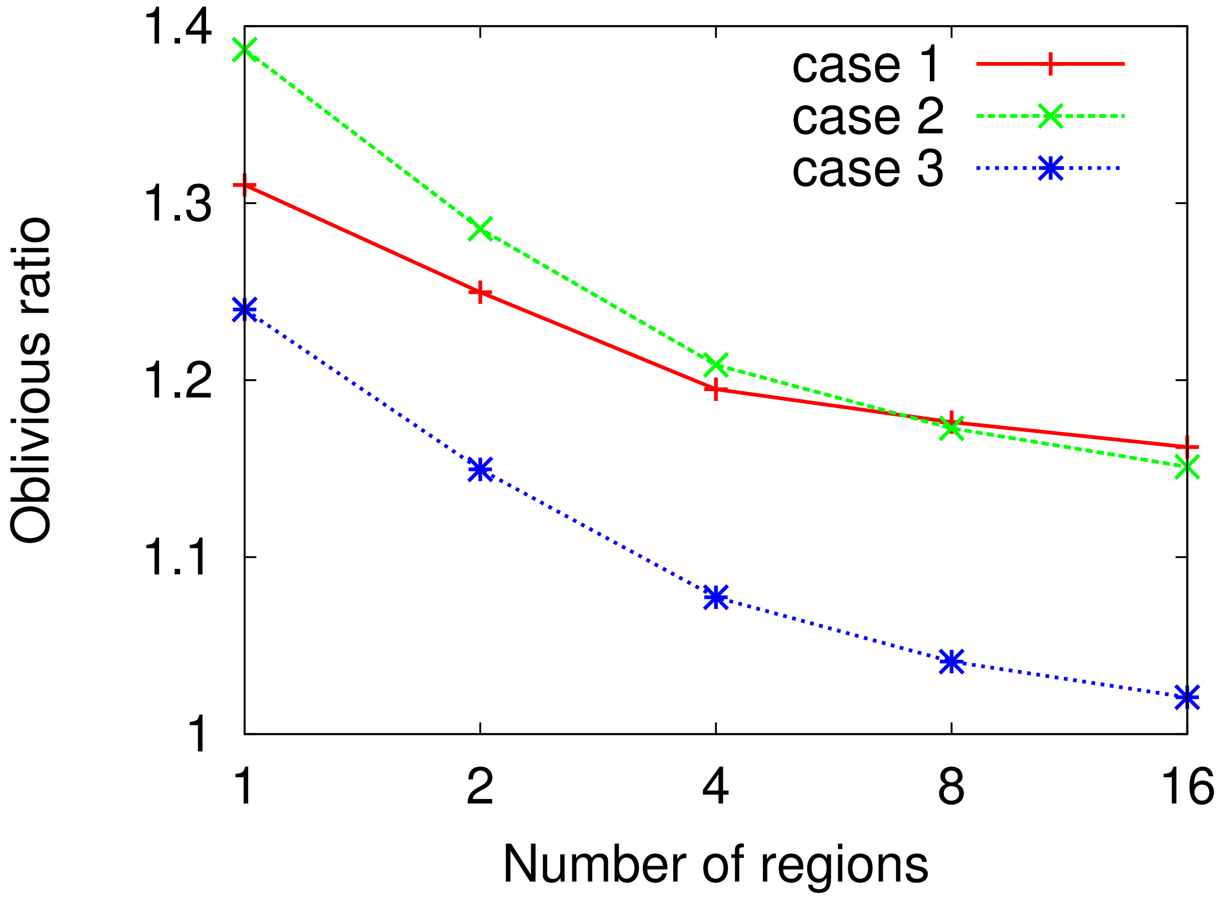
$$R_1 : \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$



$$R_2 : \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

Only a few cuts can make a difference

The oblivious ratio steadily improves as we add more cuts



Conclusions

Rate-adaptive routing: discover the distributed-centralized spectrum

Demand-oblivious routing is scalable but inefficient

We presented the first ever optimal rate-adaptive routing algorithm

- provably feasible, stable and optimizable
- heavily centralized, so hard to implement
- scales poorly

The hybrid distributed-centralized scheme seems to unify the advantages of the two