Fairness in Capacitated Networks: a Polyhedral Approach

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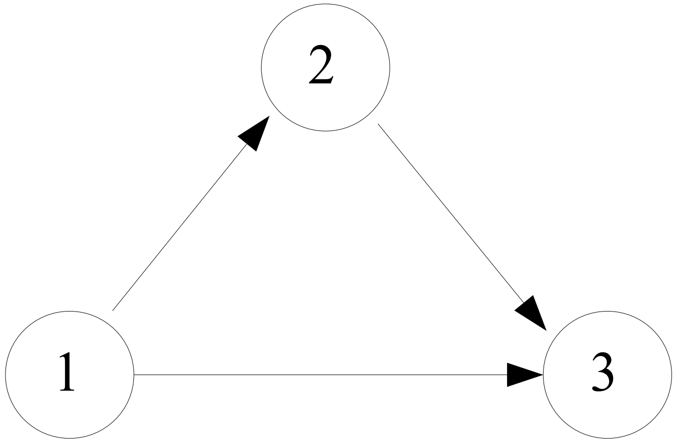
Agenda

Model: the Geometry of Networking

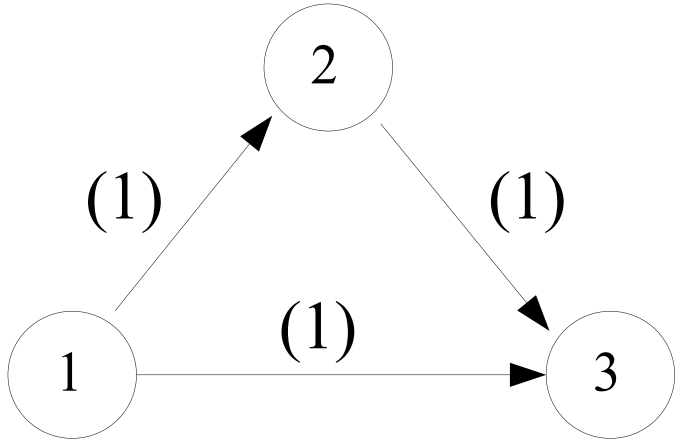
Application: fair throughput allocations in capacitated networks

A network



A graph $G(V, E)$

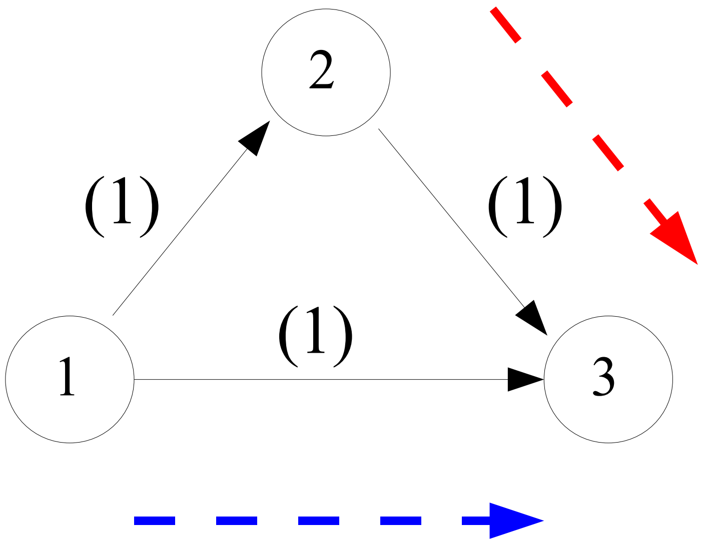
A network



A graph $G(V, E)$

Edge capacities u

A network



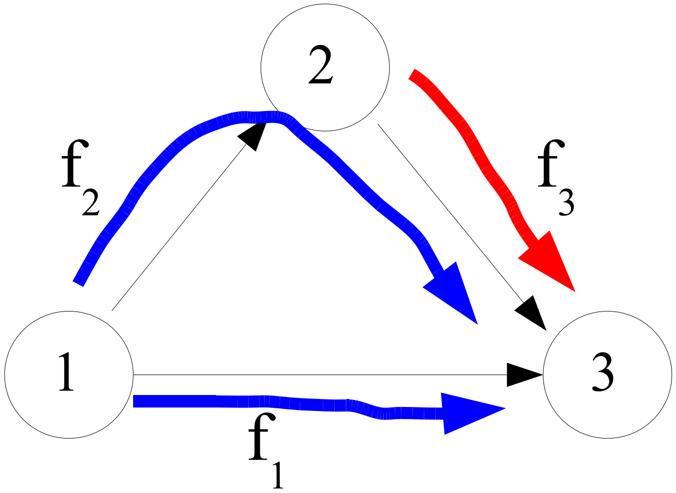
$(s_1, d_1) = (1, 3)$
$(s_2, d_2) = (2, 3)$

A graph $G(V, E)$

Edge capacities u

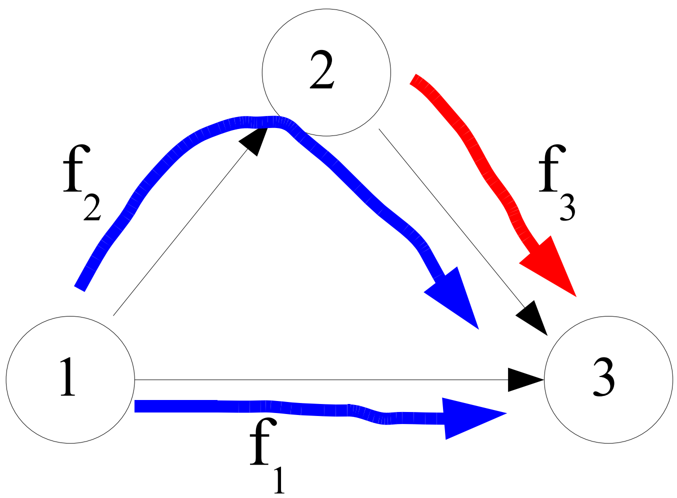
Source-destination pairs $(s_k, d_k) : k \in \mathcal{K}$

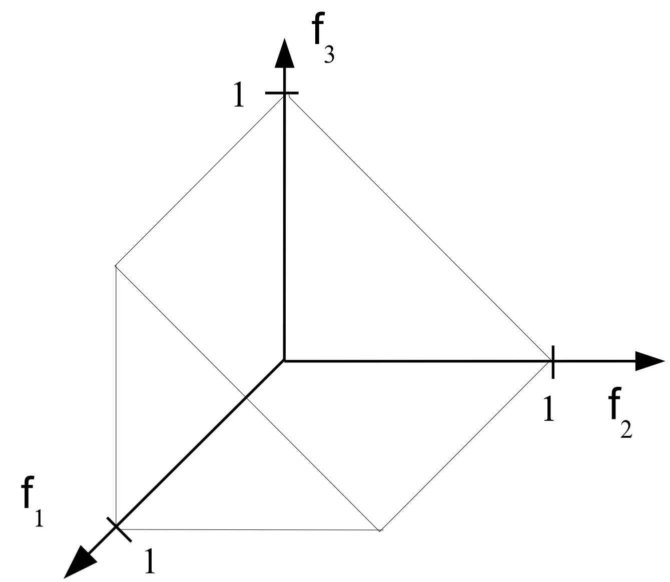
How does geometry come into the picture?



Given a network G_u

How does geometry come into the picture?

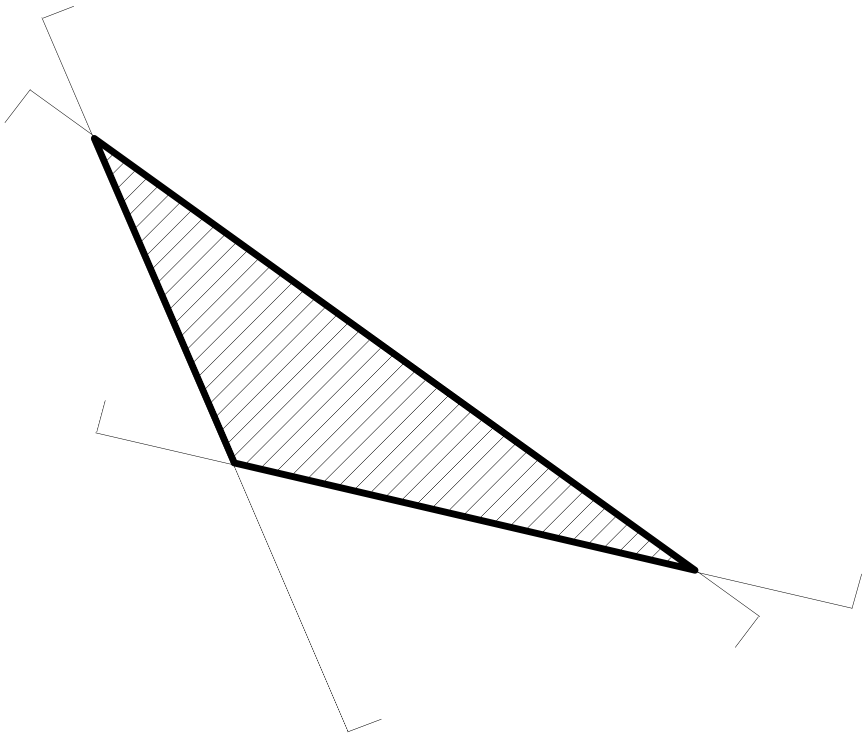




Given a network G_u

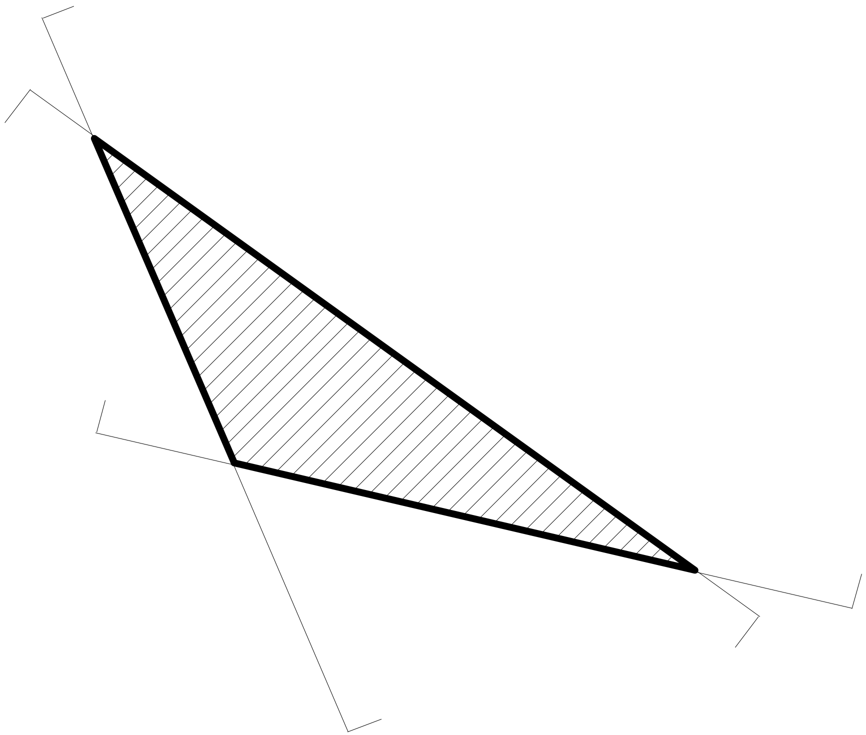
The **flow polytope** $M(G_u)$ describes all the routable path-flows

Polytopes

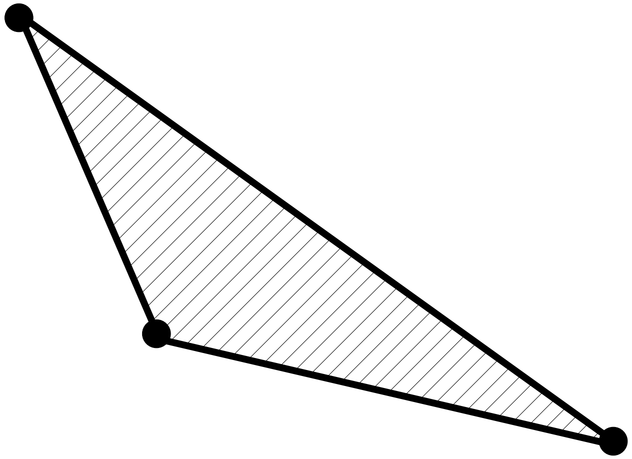


intersection of half-spaces

Polytopes



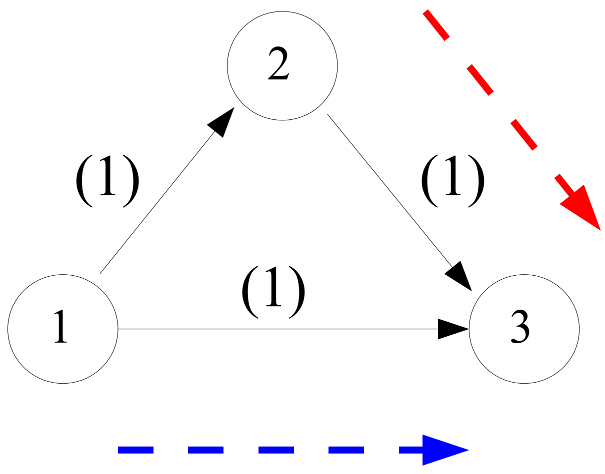
intersection of half-spaces



convex combination of points

The throughput polytope

G_u



The throughput polytope

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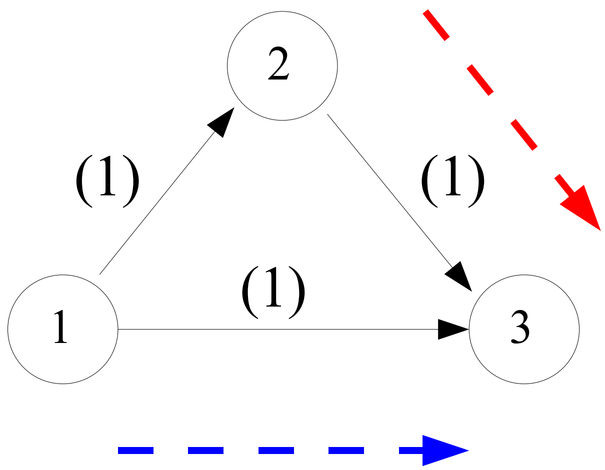
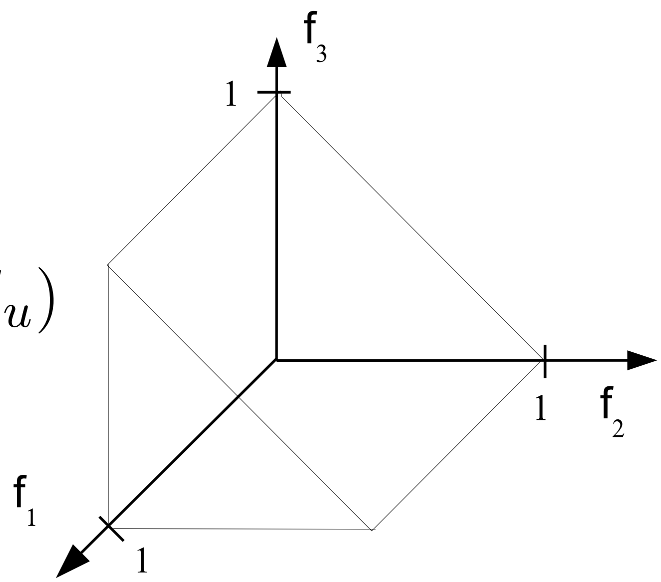
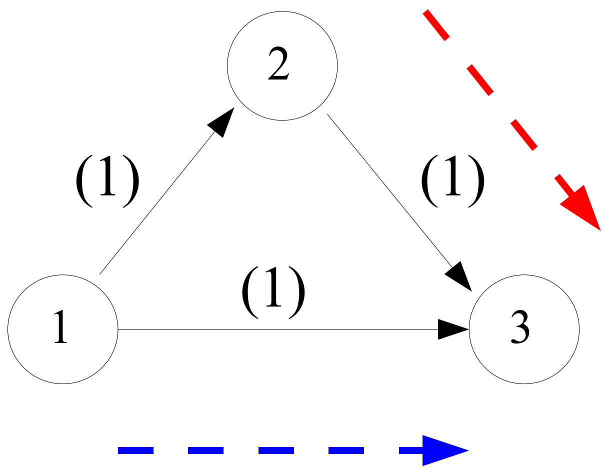


$M(G_u)$



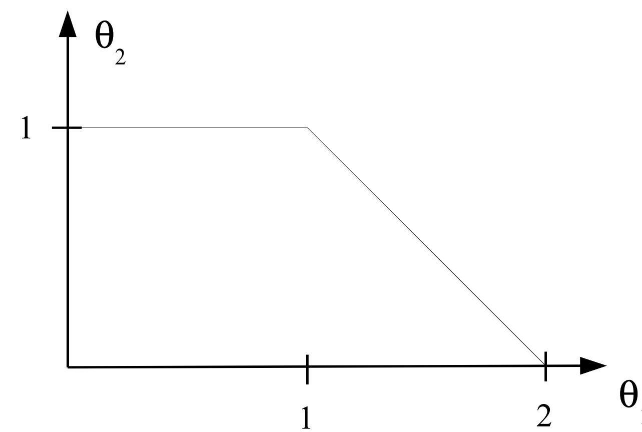
The throughput polytope

G_u



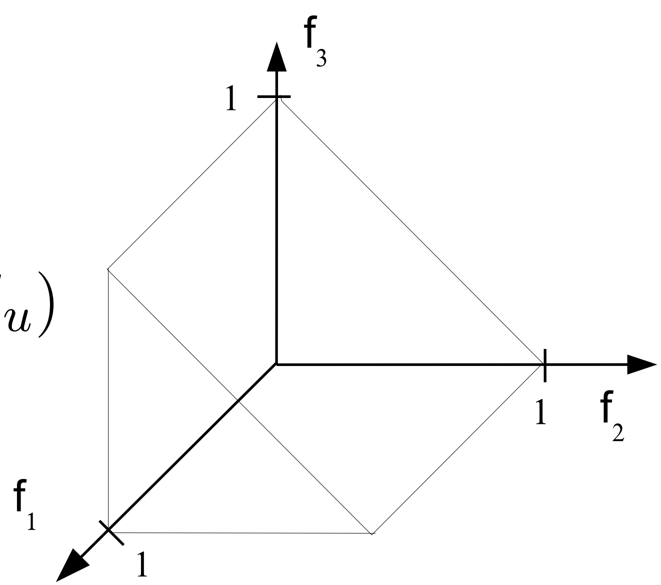
$$\theta_1 = f_1 + f_2$$

$$\theta_2 = f_3$$



$T(G_u)$

$M(G_u)$



Properties of $T(G_u)$

"The set of traffic matrices realizable in G_u"

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Polytope

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Polytope

Full-dimensional

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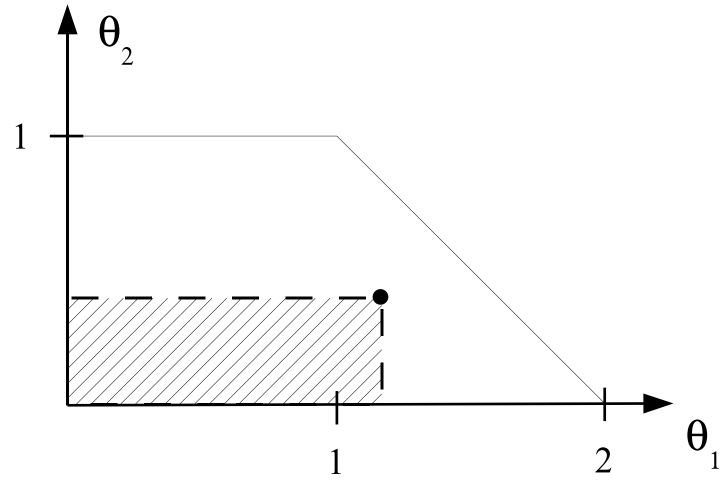
Polytope

Full-dimensional

Down-monotone

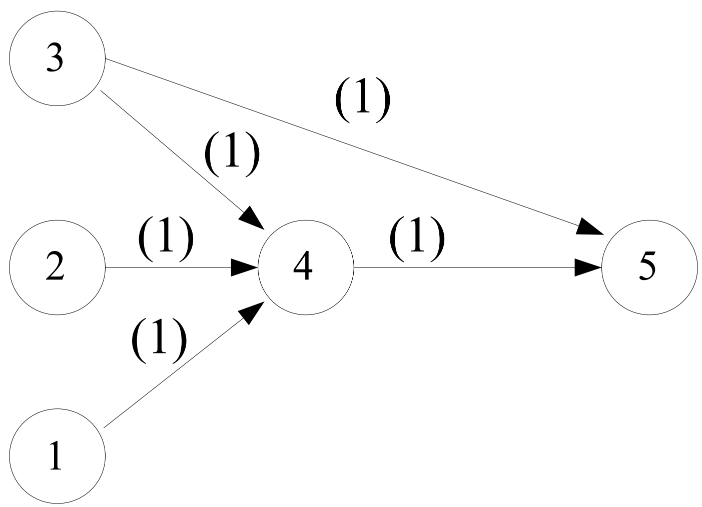


Another network

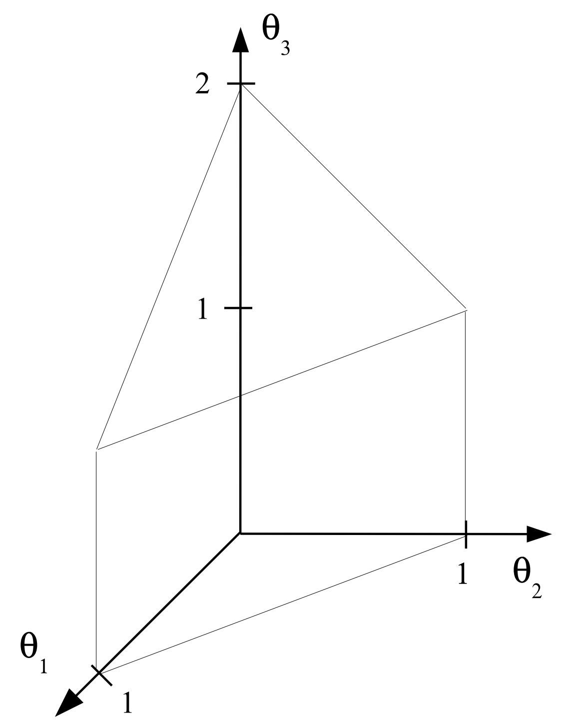
$$(s_1, d_1) = (1, 5)$$

$$(s_2, d_2) = (2, 5)$$

$$(s_3, d_3) = (3, 5)$$



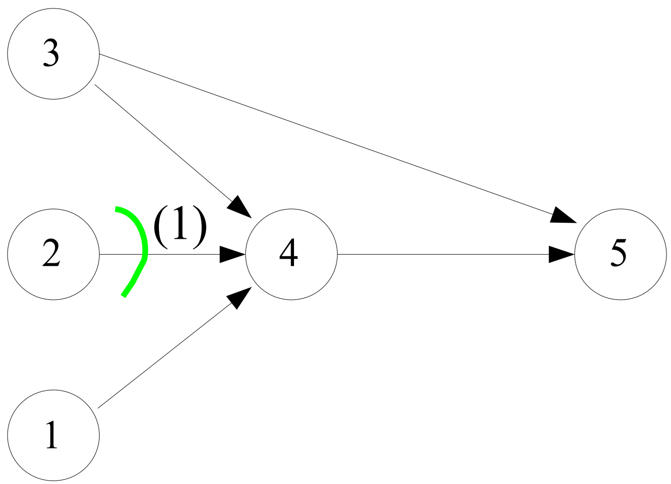
G_u



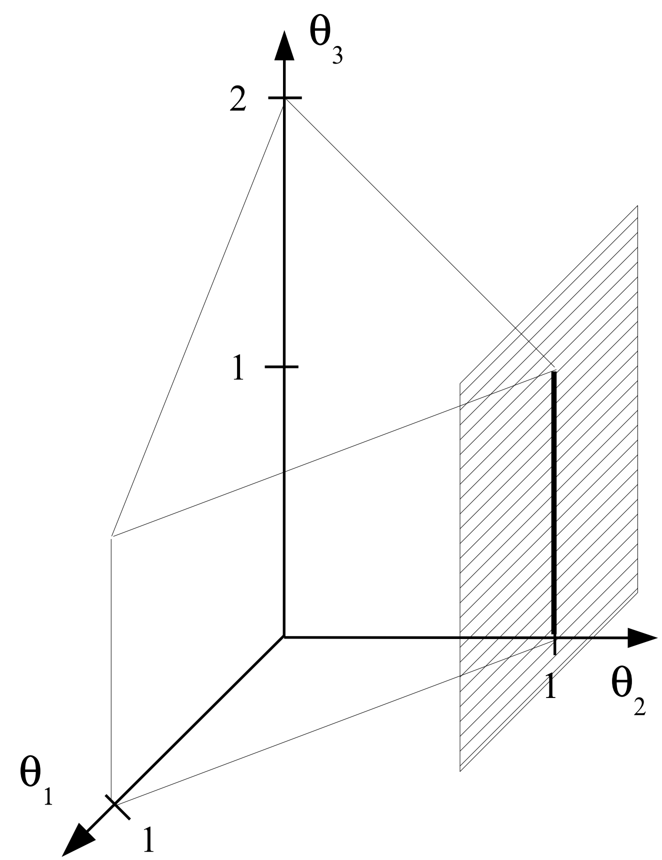
$T(G_u)$

Minimum cuts (in the Ford-Fulkerson-sense)

$(s_2, d_2) = (2, 5)$

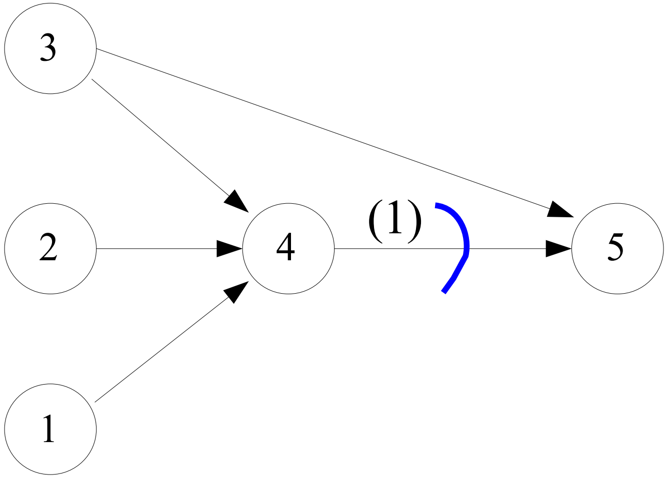


maximum flow $=$
minimum capacity cut

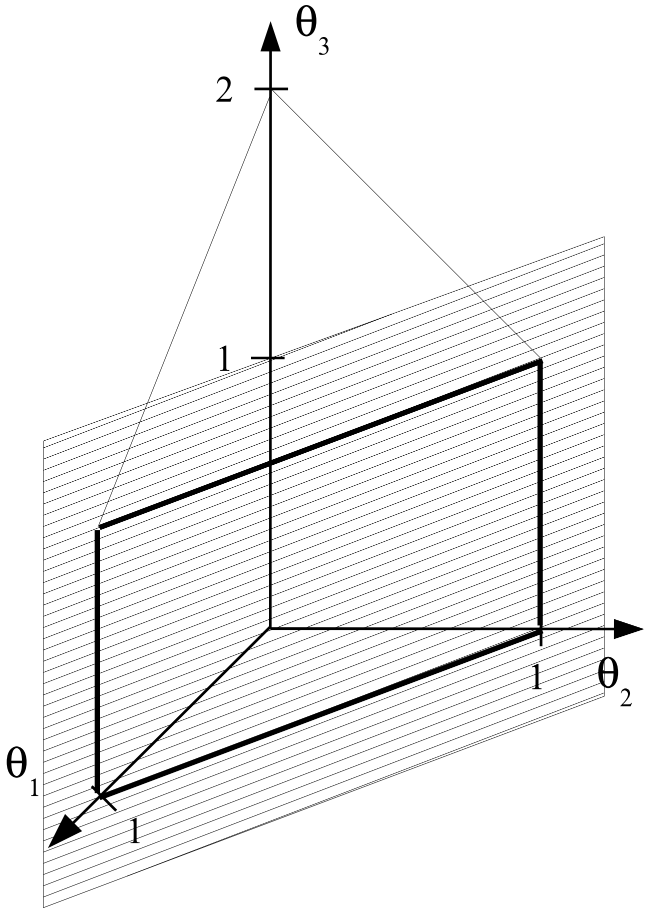


$\theta_2 \leq 1$

Minimum cuts (in the multicommodity-sense)



separating edges of
minimal capacity



$$\theta_1 + \theta_2 \leq 1$$

Fairness in capacitated networks

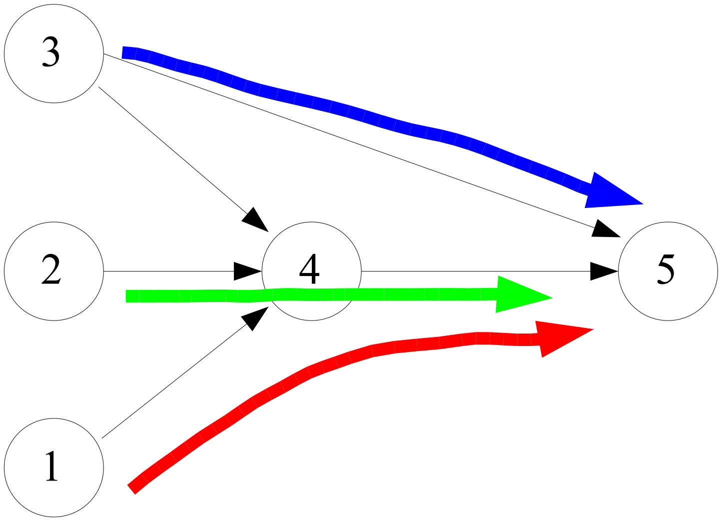
An allocation of user throughputs that is

– realizable

– efficient

– rightful

Challenge: solve this problem without having to fix the paths



$$\theta = [\tfrac{1}{2}, \tfrac{1}{2}, 1]$$

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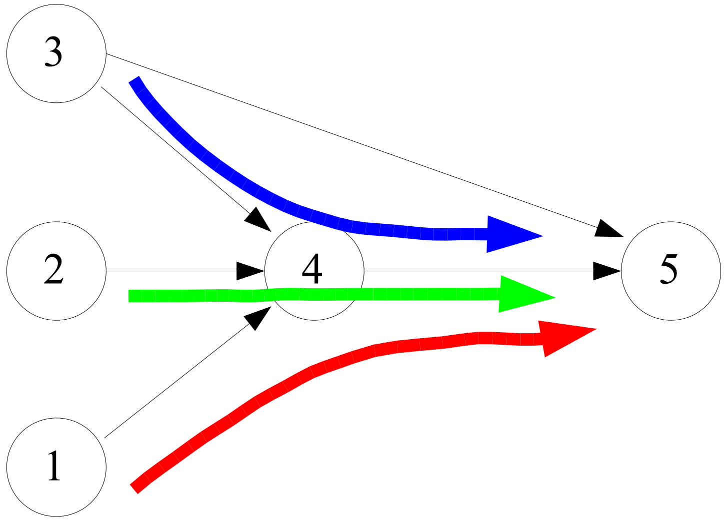

$$\theta = [\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}]$$

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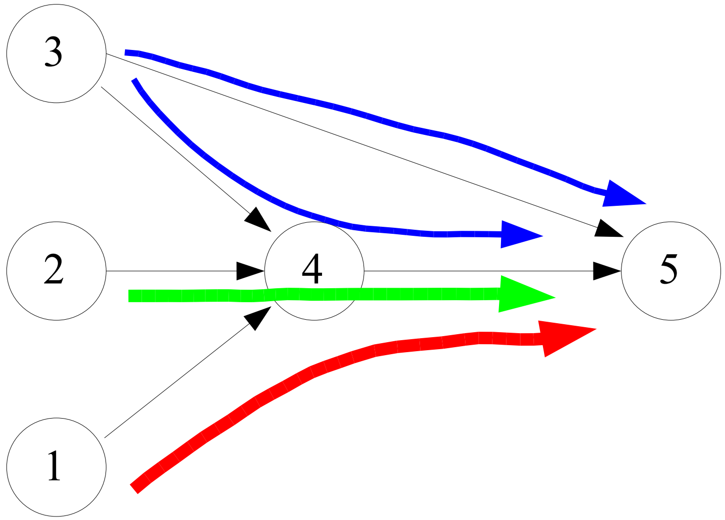


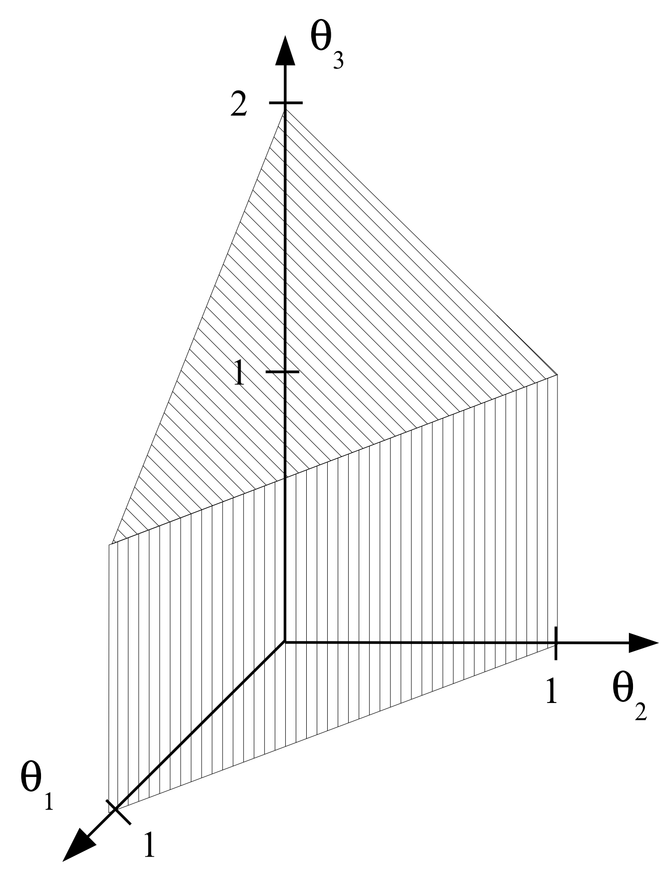
$$\theta = [\tfrac{2}{5}, \tfrac{2}{5}, \tfrac{2}{5}]$$

Efficient allocations (Non-dominatedness)

Definition: at least one user is blocked

Location: at the boundary

Problem: too wide a definition

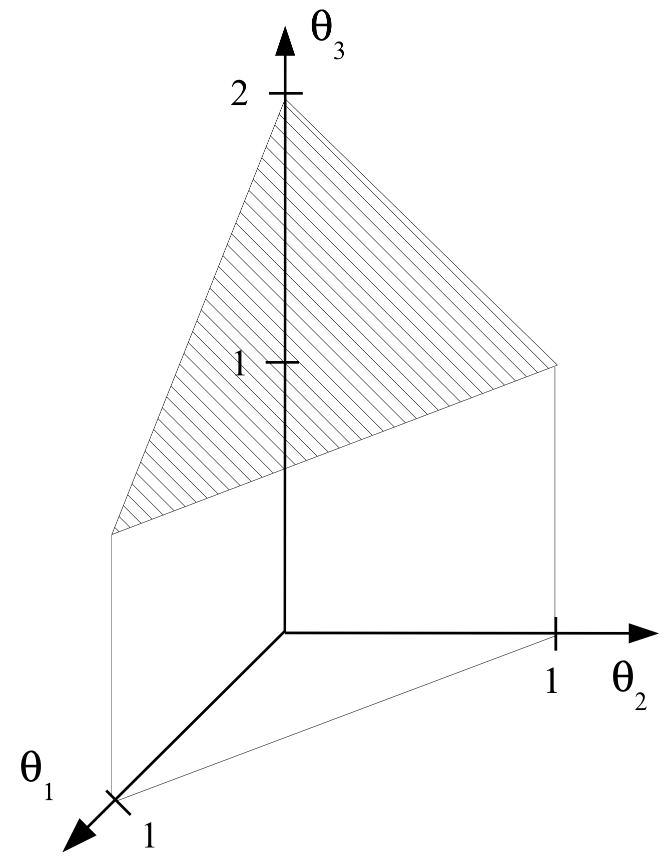


Efficient allocations (Pareto-efficiency)

Definition: no way to make any person better off without hurting anybody else

Location: at certain faces

Problem: allows for dictatorship



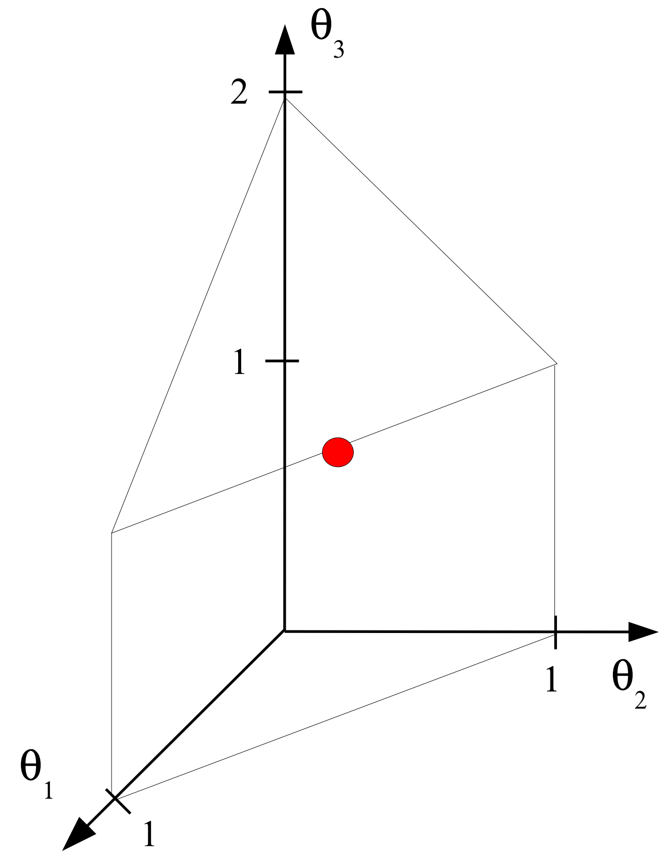
Max-min fairness

Definition: no way to make anybody better off without hurting someone else who is already poorer

– a unique max-min fair allocation exists over $T(G_u)$

– only depends on G_u

– independent of any routing whatsoever



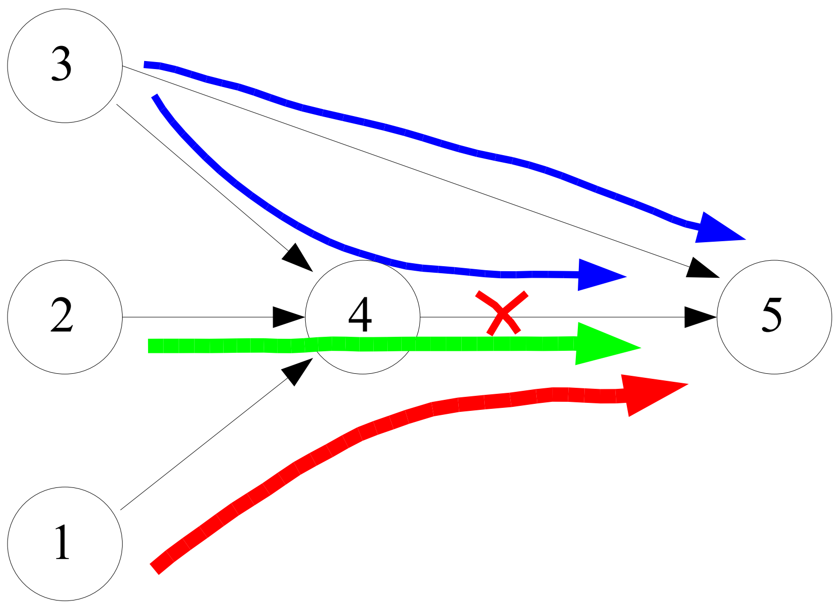
$$\theta_0 = [\tfrac{1}{2}, \tfrac{1}{2}, 1]$$

Bottlenecks (in the traditional sense)

A bottleneck edge (of some user k) is

- filled to capacity
- θ_k is maximal at the edge

Water-filling algorithm



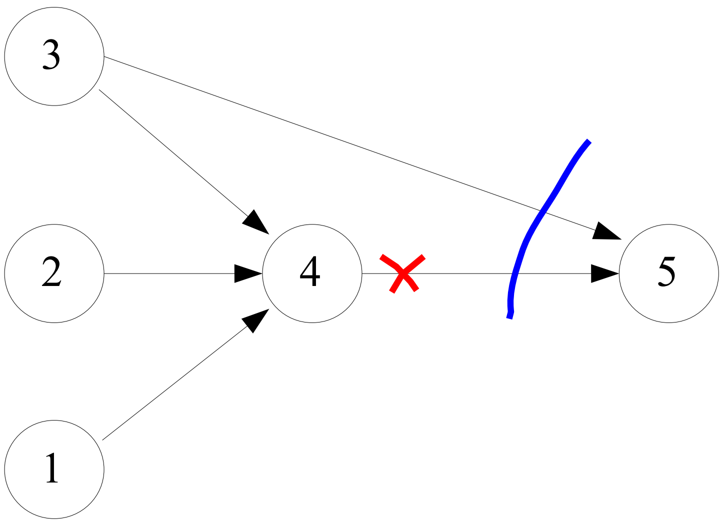
$$\theta = [\frac{2}{5}, \frac{2}{5}, \frac{2}{5}]$$

Generic bottlenecks

Geometrically: bottlenecks $\equiv$ valid inequalities

Graph-theoretically: bottlenecks $\equiv$ separating edge sets

- filled to capacity by any routing
- θ_k is maximal

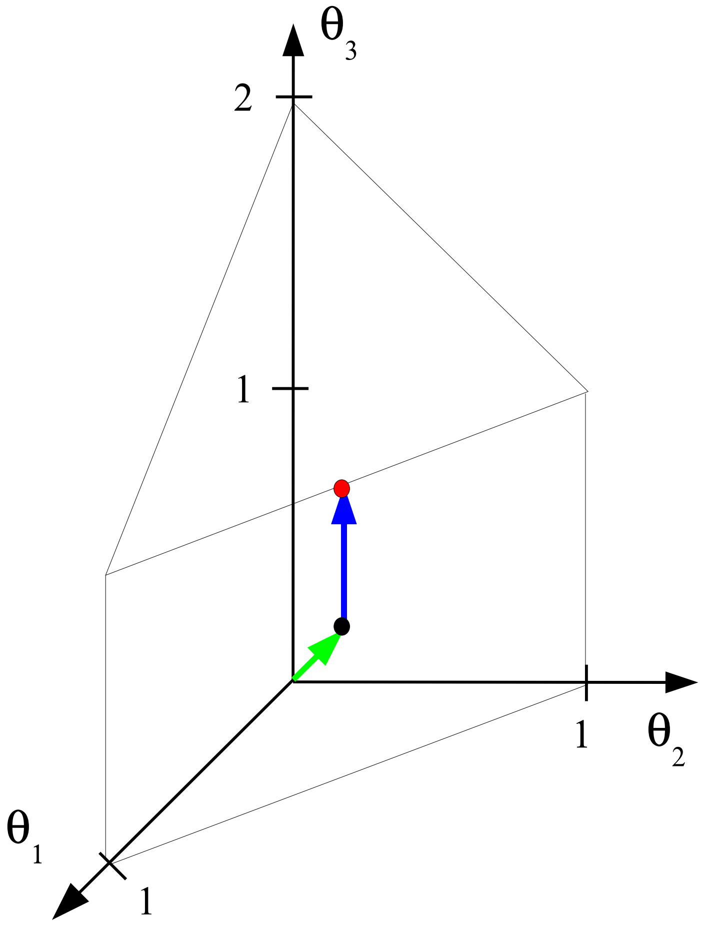


$$\theta_0 = [\tfrac{1}{2}, \tfrac{1}{2}, 1]$$

Water-filling

Find at least one bottleneck in each iteration

- start along the ray $\theta = [1, 1, 1]$
- proceed until blocked
- continue along non-blocked users



$$\theta_0 = [\tfrac{1}{2}, \tfrac{1}{2}, 1]$$

Conclusions

Geometry of Networking

– flow-theoretic reasoning

– geometric argumentation

Network fairness: a side-product

– routing-independent max-min fair allocation

– exists and unique

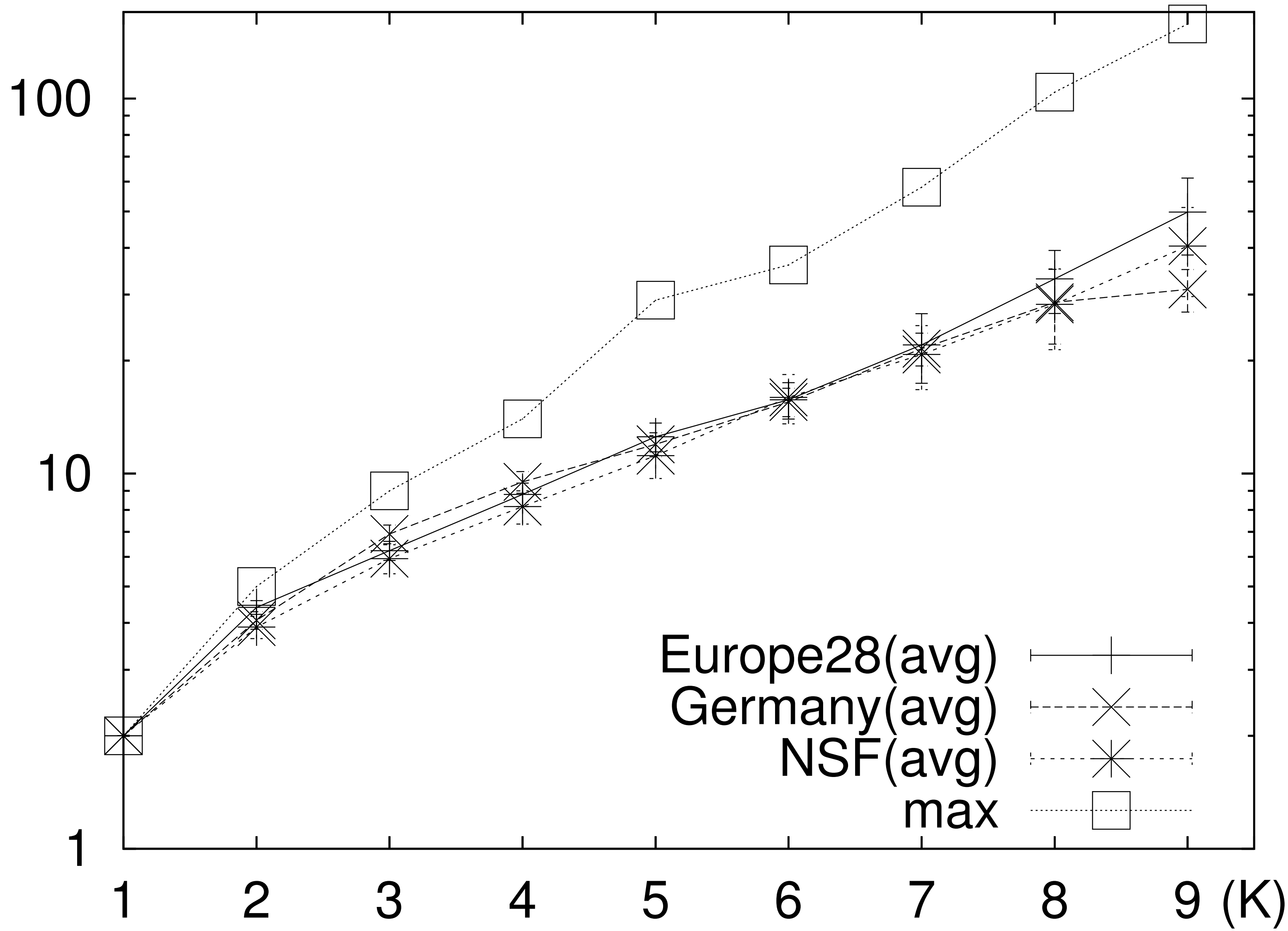
– a bottleneck argumentation (in fact, 2 ones)

– water-filling

How to compute $T(G_u)$?

– ray-shooting

Limitations



Further applications

State aggregation for inter-domain traffic engineering

- hides topological information
- reveals just enough detail

Admission control

Routing

Network decomposition