Applied Optimization and Game Theory: Syllabus

- **Introduction to Linear Programming.** Modeling with linear programs, terminology, model assumptions. The general form of linear programs, matrix notation. Maximization and minimization and conversion between the two. Types of constraints and conversion. Examples: optimal resource allocation, the transshipment problem, single- and multicommodity flow problems, portfolio design. Linear algebra: basics. Systems of linear equations, the basis and the basic solution.
- **Solving Linear Programs.** Introduction to convex analysis: convexity, convex combination, hyperplanes, half-spaces, extreme points. Convex and concave functions, the gradient, local and global optima, the Fundamental Theorem of Convex Programming (claim and proof). Convex geometry: polyhedra, the Minkowski-Weyl theorem (claim). Solving linear programs using the Minkowski-Weyl theorem, the relation of optimal feasible solutions and extreme points (claim and proof). Solving simple linear programs with the graphical method. The feasible region (bounded, unbounded, empty) and optimal solutions (unique, alternative, unbounded).
- **The simplex method.** The basics: basic solutions, basic feasible solutions, and degenerate basic feasible solutions. Iteration of the simplex method: the initial basic feasible solution, the linear program in the nonbasic variable space (development of the formula), the entering and leaving variables, the pivot. Termination: termination with optimality and with unbounded optimal solutions.
- **The simplex tableau.** Summary: the steps of the simplex method. Degeneration and cycling. Complexity (worst-case and practical). The simplex tableau. Solving linear programs using the simplex tableau: examples.
- **Duality in Linear Programming.** The dual of a linear program: motivation. Formal definition, the Karush-Kuhn-Tucker (KKT) conditions. Primal–dual relationships, the Weak and the Strong Duality Theorems (claims and proofs). The Farkas Lemma.
- **The dual simplex method.** Primal optimal (dual feasible) and primal feasible (dual optimal) bases. The dual simplex tableau, dual optimality and the dual pivot rules. Classical applications of linear programming: the use of the primal and the dual simplex methods, examples.
- **The Simplex Method: Starting Solution and Analysis.** Finding an initial basic feasible solution: the Artificial Variable technique. Sensitivity analysis: the effect of changing the objective function. Parametric analysis: perturbation of the Right-Hand-Side.
- **Classical Applications.** Optimal Product Mix and the Resource Allocation problem. Generalized assignment problem: continuous case, machine scheduling. Optimal portfolio: detecting arbitrage opportunities.
- **Nonlinear Programming 1.** General form of nonlinear programs, constrained and unconstrained optimization, convex programs, local and global optimal for nonconvex feasible region and/or objective function, complexity. Optimality conditions, smoothness, the concept of improving directions and improving feasible directions, characterizing the optimality of convex programs in terms of improving feasible directions. Solving simple nonlinear programs using successive linear programming, the Method of Zoutendijk, finding improving feasible directions, line search problems, choosing the step size.
- **Nonlinear Programming 2.** Unconstrained Optimization 1: line search, search interval, extreme values of convex functions, smooth and nonsmooth line search. Unconstrained Optimization 2: multivariable unconstrained optimization, steepest descent method. Solving constrained nonlinear using unconstrained optimization: exterior penalty function methods, (interior) barrier function methods, choosing penalty and barrier functions, comparison, examples.