Classical Applications: A Summary

WARNING: this is just a summary of the material covered in the full slide-deck ***Classical Applications*** *that will orient you as per the topics covered there; you are required to learn the full version, not just this summary!*

- Optimal Product Mix and the Resource Allocation problem
- Generalized assignment problems: continuous case, machine scheduling
- Optimal portfolio: detecting arbitrage opportunities (optional)
- Make sure you study and solve all the examples in the full-slide deck!

Optimal Product Mix

- Linear programming permeates the entire fields of economic studies, management science, operations research, and logistics
- Perhaps the most prevalent example is the Product Mix Optimization (or Production Planning) problem
- A factory has finite resources available to manufacture goods/commodities
- Products can be sold to realize immediate profit or stocked in the hope for future profit
- Goal is to determine the optimal allocation of resources in order the maximize profit
- Assumptions:
  - products and market demands are independent
  - resources are divisible
  - prices and demands can be reliably predicted

Optimal Product Mix

- The quantity of goods produced in a period plus the stock must cover the demand in each period, subject to limits on resource availability
- Goal: maximize profit while minimizing storage costs
- Let the starting stock be $y_{i0} = 0$ for each i

$$
\begin{aligned}
\max \quad & \sum_{t=1}^{T} \sum_{i=1}^{I} c_{it} \left(y_{i,t-1} + x_{it} - y_{it} \right) - \sum_{t=1}^{T} \sum_{i=1}^{I} q_{it} y_{it} \\
\text{s.t.} \quad & y_{i,t-1} + x_{it} - y_{it} = d_{it} && i \in \{1, \ldots, I\}, t \in \{1, \ldots, T\} \\
& \sum_{i=1}^{I} a_{ik} x_{it} \leq b_{kt} && k \in \{1, \ldots, K\}, t \in \{1, \ldots, T\} \\
& x_{it}, y_{it} \geq 0 && i \in \{1, \ldots, I\}, t \in \{1, \ldots, T\}
\end{aligned}
$$

Generalized Assignment Problem

- Another crucial application of linear programming in operations research
- Given m agents and n jobs that must be assigned to agents
- Each task can be performed by each agent, but with different effectiveness
  - agent i can do job j with w_{ij} units of effort, meanwhile we realize p_{ij} profits
  - agent i has w_i units of working capacity
  - b_j units of job j must be done
- Goal is to maximize profits
- Suppose that jobs are arbitrarily divisible
- Otherwise we get an integer linear program: NP-hard

Generalized Assignment Problem

- Let x_{ij} denote the quantity of job j performed by agent i [units]

$$
\begin{aligned}
\max \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij} & \\
\sum_{j=1}^{n} w_{ij} x_{ij} \le w_i & \qquad i \in \{1, \ldots, m\} \\
\sum_{i=1}^{m} x_{ij} = b_j & \qquad j \in \{1, \ldots, n\} \\
x_{ij} \ge 0 & \qquad i \in \{1, \ldots, m\}, \\
& \qquad j \in \{1, \ldots, n\}
\end{aligned}
$$